Analysis of Impact Force Equations
By Chuck Weber

This paper compares the actual impact forces measured during controlled testing to the values calculated by a commonly accepted rope force-predicting equation. All tests studied were simple "free-fall" drops where one end of the rope was secured to a rigid anchor and the other tied to the test weight – no belay devices, rope slippage at anchor or belay device, intermediate anchors, friction, etc. to factor in.
Through this study it can be shown that the universally accepted manner of modeling life-safety ropes as springs (along with the corresponding assumptions needed to allow use of the same equations) is quite accurate when the test weights and fall factors are relatively low. However, the force predictions of the equation become noticeably less accurate as the test weights and fall factors increase – as much as 30% low in certain cases.

About the Presenter

Stephen Attaway has been an active member of Albuquerque Mountain Rescue Council (AMRC), for 15 1/2 years and has participated in over 180 rescue missions. Steve's interest in rescue originated from cave exploring. Steve has been involved with cave exploration for 28 years and is a Fellow of the National Speleological Society. He participated in over 10 cave rescues including the 96-hour rescue in Lechuguilla Cave during April 1991. Steve attended Georgia Tech where he received both his Master of Science in Civil Engineering and his Ph.D. in Computational Mechanics. Competition Mechanics is the field of science related to numerical modeling of the stresses and strains associated with displacement of materials. Steve currently is a researcher at Sandia National Laboratories, where he currently holds the rank of Distinguished Member of Technical Staff in the Engineering Science and Mechanics Department. Steve has received international recognition for advances in parallel computer algorithms for computational mechanics. He is a Fellow of the American Society of Mechanical Engineers. In his current research, he predicts the response of buildings to terrorist attacks. Steve's other hobbies include goldsmithing and gemstone cutting.

Analysis of Impact Force Equations

Prepared for the International Technical Rescue Symposium,
November 2002

By Chuck Weber

Abstract

This paper compares the actual impact forces measured during controlled testing to the values calculated by a commonly accepted rope force-predicting equation. All tests studied were simple "free-fall" drops where one end of the rope was secured to a rigid anchor and the other tied to the test weight – no belay devices, rope slippage at anchor or belay device, intermediate anchors, friction, etc. to factor in.

Through this study it can be shown that the universally accepted manner of modeling life-safety ropes as springs (along with the corresponding assumptions needed to allow use of the same equations) is quite accurate when the test weights and fall factors are relatively low. However, the force predictions of the equation become noticeably less accurate as the test weights and fall factors increase – as much as 30% low in certain cases.

Data Sets

The two data sets to which this paper applied the force-predicting equation were taken directly from recent past ITRS reports:
1) *Fall Factors and Life Safety Ropes: a closer look,* ITRS 2001
2) *UIAA Dynamic Rope drop testing results with loads greater than 80 kg,* ITRS 1999.
Details on how those tests were conducted are explained in those papers, but not here.

Force-predicting equation

This paper also does not go into any great detail about all the assumptions and principles used to create force-predicting equations. However, for anyone interested in further reading or study, Steve Attaway's 1996 paper [1] does a particularly thorough and good job of explaining this issue.

Generally speaking, all 4 sources reviewed and referenced at the end of this paper used basically the same assumptions and principles to end up with a force-predicting equation that is equal to this form when the needed conversion of units or terms is applied:

$$F = W + W\sqrt{1 + \frac{2 \cdot h \cdot M}{W \cdot L}}$$

Where F = Force (in pounds of force) that will be generated when these 4 known values are plugged into the equation:

W = weight (of falling object – in pounds)
h = height of fall (in feet)
M = modulus of rope (in pounds)
L = total length of rope (in feet)

The W, h, and L values are basically self-explanatory and independent of the type of rope used, so let's look at how different values of M affect the formula's outcome.

The columns in the left half of the following table are taken from test paper #1 for all the drop tests made on PMI 12.mm Classic Static rope. The columns of the right half show the results when two different values are used for M.

PMI 12.5mm Classic Static Rope

drop ref. #	test weight (lb)	FF	drop height **h** (ft)	rope length **L** (ft)	**Force** meas. in test **F** (lbf)	*Rope Failure?* *YES*	*NO*	meas. force as % of knotted strength	used M = 15000 **FORCE** calc. by formula F (lbf)	ratio to actual	**Rating** EXCELLENT (+/- 2%)	GOOD (+/- 10%)	used M = 40000 **FORCE** calc. by formula F (lbf)	ratio to actual	**Rating** EXCELLENT (+/- 2%)	GOOD (+/- 10%)
89	176	**0.25**	1	4	985		*x*	14.5%	1338	**1.359**			2060	2.092		
90	(80 kg)	0.25	2	8	1199		*x*	17.6%	1338	**1.116**			2060	1.718		
91		0.25	4	16	1407		*x*	20.7%	1338	**0.951**		X	2060	1.464		
92		0.25	5	20	1383		*x*	20.3%	1338	**0.968**		X	2060	1.490		
131		0.25	5	20	1292		*x*	19.0%	1338	1.036		X	2060	1.595		
31	500	**0.25**	1	4	2285		*x*	33.6%	2500	1.094		X	3702	1.620		
32	(227 kg)	0.25	2	8	2663		*x*	39.2%	2500	**0.939**		X	3702	1.390		
33		0.25	4	16	3077		*x*	45.3%	2500	0.812			3702	1.203		
34		0.25	5	20	3131		*x*	46.0%	2500	0.798			3702	1.182		
146		0.25	5	20	2917		*x*	42.9%	2500	0.857			3702	1.269		
93	176	**0.5**	1	2	1091		*x*	16.0%	1810	1.659			2835	2.599		
94	(80 kg)	0.5	2	4	1413		*x*	20.8%	1810	1.281			2835	2.006		
153		0.5	4	8	1788		*x*	26.3%	1810	1.012	X		2835	1.586		
96		0.5	8	16	2007		*x*	29.5%	1810	0.902		X	2835	1.413		
97		0.5	10	20	2180		*x*	32.1%	1810	0.830			2835	1.301		
132		0.5	10	20	2046		*x*	30.1%	1810	0.885			2835	1.386		
35	500	**0.5**	1	2	2741		*x*	40.3%	3284	1.198			5000	1.824		
36	(227 kg)	0.5	2	4	3248		*x*	47.8%	3284	1.011	X		5000	1.539		
37		0.5	4	8	4235		*x*	62.3%	3284	0.775			5000	1.181		
38		0.5	8	16	5000		*x*	73.5%	3284	0.657			5000	1.000	X	
39		0.5	10	20	5126		*x*	75.4%	3284	0.641			5000	0.975	X	
152		0.5	10	20	**5045**		*x*	74.2%	3284	0.641			5000	0.991	X	
98	176	**1.0**	2	2	1633		*x*	24.0%	2481	1.519			3932	2.408		
99	(80 kg)	1.0	5	5	2358		*x*	34.7%	2481	**1.052**		X	3932	1.668		
100		1.0	10	10	2961		*x*	43.5%	2481	0.838			3932	1.328		
101		1.0	20	20	3426		*x*	50.4%	2481	0.724			3932	1.148		
133		1.0	20	20	3176		*x*	46.7%	2481	0.781			3932	1.238		
40	500	**1.0**	2	2	3908		*x*	57.5%	4405	1.127			6844	1.751		
41	(227 kg)	1.0	5	5	5774		*x*	84.9%	4405	0.763			6844	1.185		
42		1.0	10	10	**6136**	*x*		**90.2%**	**4405**	**0.718**			6844	1.115		
102	176	**2.0**	4	2	2587		*x*	38.0%	3430	1.326			5486	2.120		
103	(80 kg)	2.0	8	4	3632		*x*	53.4%	3430	0.944		X	5486	1.510		
104		2.0	12	6	4434		*x*	65.2%	3430	0.774			5486	1.237		
105		2.0	16	8	4697		*x*	69.1%	3430	0.730			5486	1.168		
43	500	**2.0**	4	2	6382		*x*	93.9%	6000	0.940		X	9458	1.482		
44	(227 kg)	2.0	8	4	6431	*x*		**94.6%**	6000	0.933		X	9458	1.471		

Accurate predictions

If you only looked at the first two groups of 0.25 fall factor in the M = 15000 section, you would likely conclude that the formula is "reasonably accurate" (+/- 10%) given the ratios shown.

The M = 15000 was determined by dividing 300 # by the rope's elongation at that weight. The M = 40000 was determined by dividing the strength of the rope at a very high load, just before rope failure, by the elongation at that point.

Note that the tests with L values of 2, 4, and 5 actually represent an overall very stretchy section of static rope due to the fact that most of their length includes the knots at each connection end. So, it stands to reason that the force-predicting equation using an M-value of 15000 along with those lengths will **overstate** the actual force recorded during the test. This is another way to say that the ratio is greater than 1.

Another note is that if the calculated F is greater than the rope's knotted breaking strength, that is to say the formula is predicting rope failure.

You will see that essentially all of the ratios in the M = 40000 section are **overstated.** However, it is interesting to note that this M value does create very accurate force predictions when the forces recorded in the drop test are very high and near the rope's breaking strength.

Drop #42

This particular test makes a convincing argument that the formula should NOT be relied upon to predict the force generated in this fall. For the M = 15000 section the predicted F value, 4405, is much less than the knotted rope breaking strength of 6800, so one would assume that the rope would NOT fail.

However, it is critical to note that the rope DID FAIL in this test and the force recorded at the moment of failure was 6136.

The ratio in this example is useful to look at, but not 100% accurate because had the rope NOT failed, the force would have to have been a little higher. So, we can say that the **formula predicted a force AT LEAST 28% LOW.** Needless to say, this would surely be unacceptable in anyone's book. However, note that the much higher M value accurately predicts rope failure for this example.

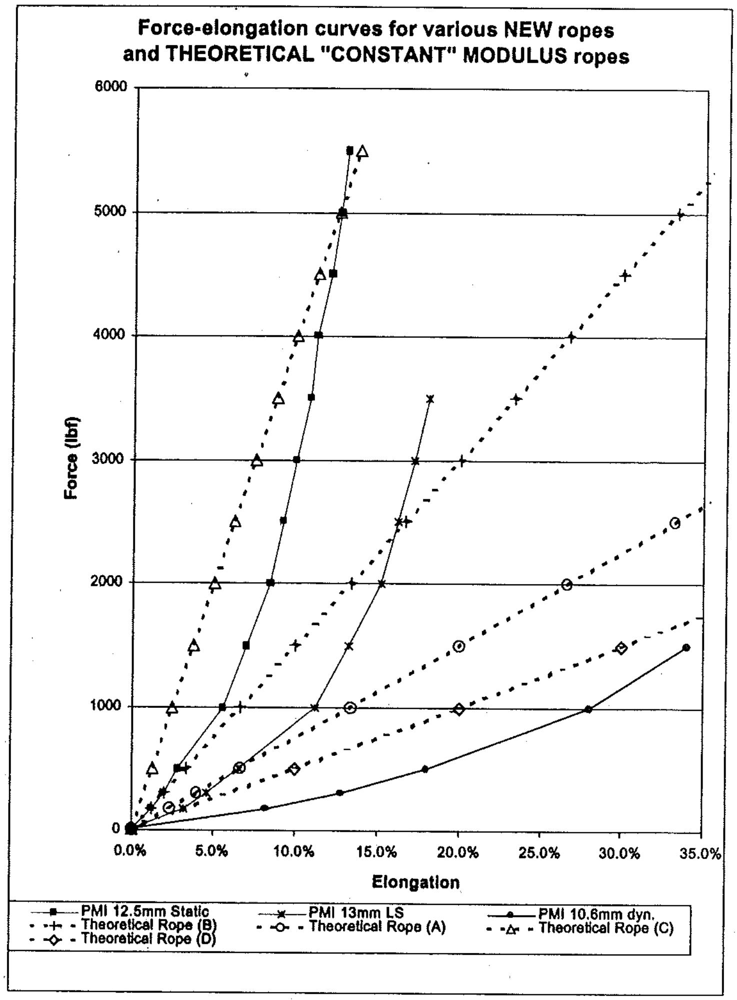
Other static rope diameters show the same trends, but are not detailed in this paper. Overall, this analysis suggests there is no single M-value that can be used in the given equation to accurately predict the force for all scenarios for static ropes.

If ropes truly acted like springs

The graph below shows the actual slow-pull test data curves for the three ropes detailed in this paper.

Added to this graph are four theoretical ropes to show what truly "linear" force-elongation curves would look like. The M-values for these theoretical ropes are:
A = 7500, B = 15000, C = 40000, D = 5000.

A primary assumption needed to create this force-predicting equation was that the force-elongation curves of static and low-stretch ropes were close enough to straight lines that the ropes' performance could be modeled by spring equations. This is what allows otherwise mathematically complex equations to be simplified down to the equation presented.



Apply same analysis to a Low-Stretch design rope

The table below applies the same method of analysis to a different type of rope. Many of the same trends are noticed. The M = 8000 is derived from a 300 # load divided by the elongation at that load.

Drop # 70

In a similar fashion to the drop #42 section before, using M = 8000 (about ½ that of the static rope) for this rope in the formula gives a force well below the rope's known knotted breaking strength (4900 #). But, the rope actually FAILED in this setup and the force recorded at the moment of failure was 4688#. Again, the formula predicted a force that was too low.

Blue Water II +Plus 7/16" (11.6mm) Low-Stretch

drop ref. #	weight (lb)	FF	drop height h (ft)	rope length L (ft)	Force meas. in test F (lbf)	Rope Failure? YES	NO	used M = 8000: FORCE calc. by formula F (lbf)	ratio to actual	EXCELLENT (+/- 2%)	GOOD (+/- 10%)	used M = 15000: FORCE calc. by formula F (lbf)	ratio to actual	EXCELLENT (+/- 2%)	GOOD (+/- 10%)
72	**176**	**0.25**	**1**	4	869		x	1033	**1.189**			1547	**1.780**		
73	(80 kg)		**2**	8	984		x	1033	**1.050**		X	1547	**1.572**		
74			**4**	16	1063		x	1033	**0.972**		X	1547	**1.455**		
75			**5**	20	1090		x	1033	**0.948**		X	1547	**1.419**		
140			**5**	20	1067		x	1033	**0.968**		X	1547	**1.450**		
45	**500**	**0.25**	**1**	4	1943		x	2000	**1.029**		X	2845	**1.464**		
46	(227 kg)		**2**	8	2307		x	2000	**0.867**			2845	**1.233**		
47			**4**	16	2519		x	2000	**0.794**			2845	**1.129**		
48			**5**	20	2595		x	2000	**0.771**			2845	**1.096**		X
149			**5**	20	2612		x	2000	**0.766**			2845	**1.089**		X
76	**176**	**0.50**	**1**	2	1062		x	1376	**1.295**			2107	**1.984**		
77	(80 kg)		**2**	4	1399		x	1376	**0.983**	X		2107	**1.506**		
78			**4**	8	1559		x	1376	**0.882**			2107	**1.351**		
79			**8**	16	1742		x	1376	**0.790**			2107	**1.209**		
80			**10**	20	1819		x	1376	**0.756**			2107	**1.158**		
141			**10**	20	1646		x	1376	**0.836**			2107	**1.280**		
49	**500**	**0.50**	**1**	2	2360		x	2562	**1.085**		X	3779	**1.601**		
50	(227 kg)		**2**	4	2964		x	2562	**0.864**			3779	**1.275**		
51			**4**	8	3704		x	2562	**0.692**			3779	**1.020**	X	
52			**8**	16	4042		x	2562	**0.634**			3779	**0.935**		X
53			**10**	20	4197		x	2562	**0.610**			3779	**0.900**		X
81	**176**	**1.00**	**1**	1	1551		x	1863	**1.201**			2901	**1.870**		
82	(80 kg)		**5**	5	2151		x	1863	**0.866**			2901	**1.348**		
83			**10**	10	2682		x	1863	**0.695**			2901	**1.081**		X
84			**20**	20	2901		x	1863	**0.642**			2901	**1.000**	X	
142			**20**	20	2605		x	1863	**0.715**			2901	**1.113**		
68	**500**	**1.00**	**2**	2	3716		x	3372	**0.908**		X	5110	**1.375**		
69	(227 kg)		**5**	5	4966		x	3372	**0.679**			5110	**1.029**		X
70			**10**	10	4688	x		3372	**0.719**			5110	**1.090**		X
85	**176**	**2.00**	**4**	2	2515		x	2556	**1.016**	X		4025	**1.600**		
86	(80 kg)		**8**	4	3367		x	2556	**0.759**			4025	**1.195**		
87			**12**	6	3846		x	2556	**0.665**			4025	**1.047**		X
88			**16**	8	4138		x	2556	**0.618**			4025	**0.973**		X
71	**500** (227 kg)	**2.00**	**4**	2	4584	x		4531	**0.988**	X		7000	**1.527**		

Dynamic Ropes

Using the data set from paper #2, the following table was created in a similar fashion as before. Again both low and high moduli were used to compare what force the equation will predict for very different M-values.

M = 2200 corresponds to the test weight, 176#, divided by its static elongation, 8%.

None of the ratios for this set were even within 10% of the actual recorded value. The formula predicted **too low a value in all cases,** but none resulted in rope failure.

M = 5250 corresponds to the rope's impact force during the first drop test, 1866#, divided by an approximate maximum dynamic elongation measured during that drop, 35.5%.

The ratio column in the table clearly shows that this M-value produced many "reasonably accurate" results for a variety of test weights and drop heights.

However, it should also be pointed out that the lowest FF shown in this table is 0.7 and it is suspected that had more tests been conducted for each test weight group, this high M value would have produced less accurate results as the FF decreased further.

PMI 10.5mm Dynamic Rope
L = 8.5 ft in all tests

					M = 2200		M = 5250			
Drop ref. #	**FF**	Test Weight (lb)	h (ft)	**Force** meas. **(lbf)**	Force calc. F (lbf)	ratio to actual	Force calc. F (lbf)	ratio to actual	EXCELLENT (+/- 2%)	GOOD (+/- 10%)
1	**1.7**	**176**	15.6	**1866**	1380	**0.739**	2024	**1.085**		X
2	1.7	200	15.6	2038	1485	**0.729**	2171	**1.065**		X
3	1.6	200	14.7	1978	1448	**0.732**	2113	**1.068**		X
4	1.5	200	13.8	1908	1409	**0.738**	2053	**1.076**		X
5	1.4	200	12.9	1830	1369	**0.748**	1991	**1.088**		X
6	1.7	225	15.6	2261	1590	**0.703**	2317	**1.025**	X	
7	1.6	225	14.7	2190	1550	**0.708**	2255	**1.030**		X
8	1.5	225	13.8	2136	1509	**0.707**	2191	**1.026**		X
9	1.4	225	12.9	2048	1467	**0.716**	2126	**1.038**		X
10	1.3	225	11.9	1925	1424	**0.740**	2058	**1.069**		X
11	1.2	225	11.0	1854	1378	**0.743**	1987	**1.072**		X
12	1.7	250	15.6	2499	1691	**0.677**	2456	**0.983**	X	
13	1.6	250	14.7	2383	1649	**0.692**	2391	**1.004**	X	
14	1.5	250	13.8	2315	1606	**0.694**	2324	**1.004**	X	
15	1.4	250	12.9	2190	1562	**0.713**	2255	**1.030**		X
16	1.3	250	11.9	2128	1516	**0.712**	2183	**1.026**		X
17	1.2	250	11.0	2039	1468	**0.720**	2109	**1.034**		X
18	1.1	250	10.1	1900	1419	**0.747**	2031	**1.069**		X
19	1.0	250	9.2	1842	1367	**0.742**	1950	**1.059**		X
20	1.7	276	15.7	2740	1797	**0.656**	2603	**0.950**		X
21	1.6	276	14.8	2632	1753	**0.666**	2535	**0.963**		X
22	1.5	276	13.9	2549	1708	**0.670**	2464	**0.967**		X
23	1.4	276	13.0	2383	1662	**0.697**	2392	**1.004**	X	
24	1.3	276	12.0	2297	1614	**0.703**	2317	**1.009**	X	
25	1.2	276	11.1	2167	1564	**0.722**	2239	**1.033**		X
26	1.1	276	10.2	2051	1512	**0.737**	2158	**1.052**		X
27	1.0	276	9.3	1915	1458	**0.761**	2073	**1.082**		X
28	0.9	276	8.4	1816	1401	**0.772**	1984	**1.092**		X
29	1.7	301	15.7	3046	1891	**0.621**	2732	**0.897**		X
30	1.6	301	14.8	2793	1846	**0.661**	2661	**0.953**		X
31	1.5	301	13.9	2686	1799	**0.670**	2588	**0.963**		X
32	1.4	301	13.0	2575	1751	**0.680**	2512	**0.976**	X	
33	1.3	301	12.0	2460	1701	**0.691**	2434	**0.989**	X	
34	1.2	301	11.1	2368	1649	**0.696**	2353	**0.993**	X	
35	1.1	301	10.2	2312	1595	**0.690**	2268	**0.981**	X	
36	1.0	301	9.3	2175	1538	**0.707**	2179	**1.002**	X	
37	0.9	301	8.4	2052	1479	**0.721**	2086	**1.017**	X	
38	0.8	301	7.4	1895	1417	**0.748**	1988	**1.049**		X
39	0.7	301	6.5	1768	1351	**0.764**	1884	**1.066**		X

It can be seen in the graph that the curve for dynamic rope does not curve upward as quickly as the other two shown. Instead, for any given force the dynamic rope elongates more (shown by curve stretching to the right), as is expected, than the others. This is what is meant by it has a lower modulus.

Results

The analysis given thus far suggests that:

M must change to make formula produce accurate results in all cases.

A low modulus was compared to a high modulus to demonstrate that no single value of M worked in all cases for any of the ropes tested. This is a fundamental conflict with the assumption that rope performance can be modeled by spring equations. If ropes could be modeled as conventional metal coiled springs then one M value would prove reasonably accurate for most cases.

This point is restated by the fact that the rope force-elongation curves do not lend themselves to being approximated by straight lines.

Approximating the force-elongation curves of all the ropes tested as straight lines so that they can be modeled as springs to predict forces is only accurate for a limited range. That range is loosely defined as the combinations of W, h, and L that produce forces near the point at which the modulus value is based.

As shown by the ratio values in the tables and the occasional rope failure result that the formula failed to predict, **it is always better to overstate the force prediction than understate.**

Aside from these limitations this formula has been around for many years, but never been "user-friendly" for everyday use in the field (unless you have the uncanny ability to do that type of math in your head).

Polyester rope

Polyester ropes were not evaluated for this paper, but if it is claimed that these ropes have less elongation than nylon "static" ropes then the force-elongation curves for those ropes are very "steep" and would lie to the left of the ropes shown in the graph. A straight line might reasonably approximate that rope's force-elongation curve and thus this formula would likely be fairly accurate in predicting forces for that rope.

Comparing rope types

Here are some suggested "rules of thumb" useful to rope users for comparing different rope types.
- always assume that a LOWER ELONGATION rope = HIGHER IMPACT FORCE in the event of a fall onto the rope
- the listed MBS on a rope gives no indication of the rope's ability to absorb energy

Is tracking a rope's modulus over time valuable for inspection purposes?

After input from ITRS attendees last year one concept that was briefly explored for this paper was whether there was any value to rope user's to measure and document a rope's elongation value (or modulus) at some frequency over its service life.

The findings are preliminary at this point, but quite interesting. Quickly said, this early analysis is not enough to say for certain whether this would be a worthwhile practice. Perhaps a future study for an interested party.

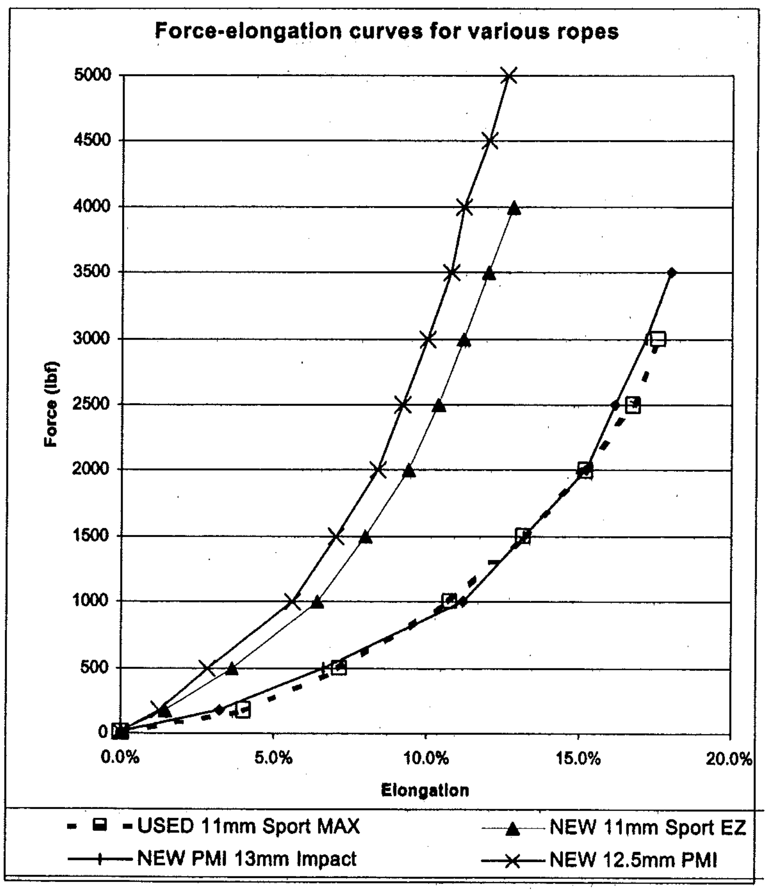
The interesting discovery thus far pertains to this. One might expect the modulus to increase as the rope becomes used and "worn out," so to speak. We all have likely heard from a reputable source at some point in time that ropes get "stiffer" over time and have less ability to absorb energy.

However, consider the graph on the following page that compares a new 11mm PMI White Classic Static rope to a 15-year-old rope of the same type.

The old rope did feel "stiffer" than the new one, but a clarification needs to be made between this use and the material science definition. The old rope feels "stiff" due to shrinkage and tightening of the white core and sheath yarns, their loss of lubricants over 15 years of very heavy use, including many speed rappels, and the fact that it intentionally started out as a stiff PMI Max-Wear rope.

In the material science sense one material is said to be "stiffer" than another one if its modulus is higher. For example, polyester is "stiffer" than nylon and static rope is "stiffer" than dynamic rope.

You can see in the graph that over 15 years of very heavy use this rope's force-elongation curve did not shift to the left, i.e. become stiffer and develop a higher modulus, but rather it shifted to the right and "flattened out" enough to basically overlap the curve of a low-stretch design rope. This says that the modulus has decreased, not increased.



However, it is important to note that this rope has "lost strength." The slow-pull knotted rope test for this 15-year-old rope showed a 33% decrease in strength to that of the new rope.

Another test performed on a new PMI 12.5mm Static Rope showed a similar phenomenon. The elongation was checked first, then a severe drop test was performed that resulted in total rope failure at one of the knots. This left a long section of the rope intact with one of the original knots. After about 1 hour of rest (no load on rope), the elongation was measured again using the exact same gauge marks as originally used. The result was similar to that shown in the graph for the 11mm rope – the elongation was noticeably increased, not decreased.

Similar testing was also done on single PMI nylon core yarn bundles. Applying many small loads did not change the elongation. However, after "damaging" loads were applied the elongation did noticeably increase – thus the modulus decreased.

Future testing suggestions
Confirm which direction of shift exists for the modulus of all rope types after use and whether it could be used as a rope inspection criterion.

References

1. Attaway, Steven, "Rope System Analysis," 1996

2. Cinnamon, Jerry, Climbing Rock and Ice, 1994

3. PETZL, Fall Simulator, www.petzl.com

4. BEAL ROPES, "Impact Force – Theory and Practice," BEAL catalog 2002